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APPLICATION OF FIBER BEAM-COLUMN ELEMENT IN DRAIN-3DX (TYPE 15)

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1 Introduction

Recently it was a common practice to analyze bridges in two horizontal directions (longitudinal and transverse) separately, making two plane models. However, it was realized that 3D analysis is necessary for better estimation of bridge response.

In the 3D analysis, just like in the 2D analysis, different models of structure can be defined, in the range from very simple lumped models to very sophisticated finite element models. One of the parameters, which can influence the level of model simplification is the type of the analysis. Usually, for the nonlinear dynamic analysis simplified structural components models are defined. In those models, structure is discretized into certain number of elements, where the response of each element is defined with different hysteresis rules. However, in more sophisticated modeling, fiber element model can also be used. Since it was expected that such a model could result in the better estimation of the bridge response, the study of this element was performed, using program Drain-3DX.

Within this study, the non-linear dynamic analysis of two structures was performed:

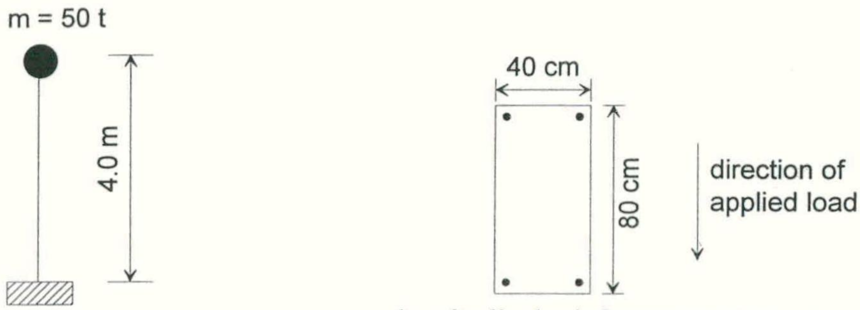
- a simple cantilever column, and
- a four-span single-column-bent viaduct, which has been originally used in the experimental and analytical studies in support of the European standard for seismic design of bridges - EC8/2 (Pinto A. V. 1996; Calvi G.M. and Pinto P.E. 1996).

Both structures were analyzed using 3D models, but seismic load was applied only in the transverse direction, first. Although the complete 3D analysis (considering two horizontal components of seismic load) was planned, it was not performed, since the results of the first analysis were not as good as it was expected.

2 Description of Analyzed Structures

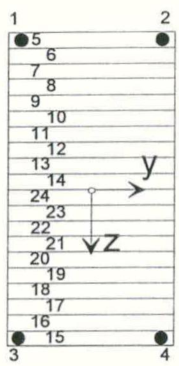
Cantilever column

Analysis of the element type 15 was performed on two types of structures. First structure was a simple cantilever column, with the rectangular cross-section. All the data used in modeling of this structure are presented in Figures 1, 2 and 3.



longitudinal reinforcement percentage is 1%

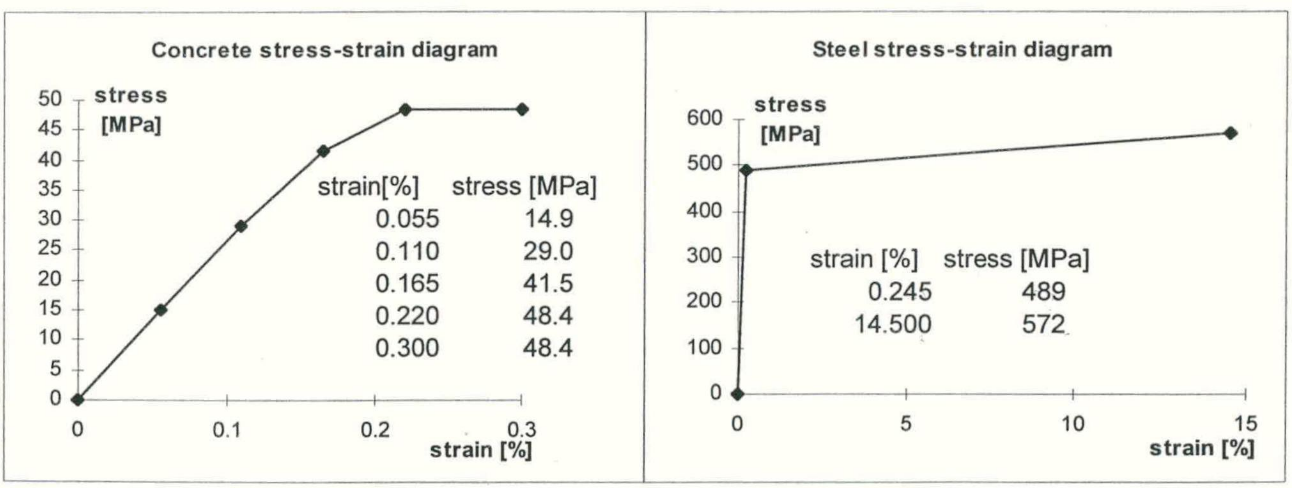
Figure 1. Cantilever column - geometry



fib	y-coor. [m]	z-coor. [m]	area [m^2]	type	fib	y-coor. [m]	z-coor. [m]	area [m^2]	type
1	-0.18	-0.38	$8 \cdot 10^{-4}$	S*	13	0.00	-0.06	$16 \cdot 10^{-3}$	C
2	0.18	-0.38	$8 \cdot 10^{-4}$	S	14	0.00	-0.02	$16 \cdot 10^{-3}$	C
3	-0.18	0.38	$8 \cdot 10^{-4}$	S	15	0.00	0.38	$16 \cdot 10^{-3}$	C
4	0.18	0.38	$8 \cdot 10^{-4}$	S	16	0.00	0.34	$16 \cdot 10^{-3}$	C
5	0.00	-0.38	$16 \cdot 10^{-3}$	C*	17	0.00	0.30	$16 \cdot 10^{-3}$	C
6	0.00	-0.34	$16 \cdot 10^{-3}$	C	18	0.00	0.26	$16 \cdot 10^{-3}$	C
7	0.00	-0.30	$16 \cdot 10^{-3}$	C	19	0.00	0.22	$16 \cdot 10^{-3}$	C
8	0.00	-0.26	$16 \cdot 10^{-3}$	C	20	0.00	0.18	$16 \cdot 10^{-3}$	C
9	0.00	-0.22	$16 \cdot 10^{-3}$	C	21	0.00	0.14	$16 \cdot 10^{-3}$	C
10	0.00	-0.18	$16 \cdot 10^{-3}$	C	22	0.00	0.10	$16 \cdot 10^{-3}$	C
11	0.00	-0.14	$16 \cdot 10^{-3}$	C	23	0.00	0.06	$16 \cdot 10^{-3}$	C
12	0.00	-0.10	$16 \cdot 10^{-3}$	C	24	0.00	0.02	$16 \cdot 10^{-3}$	C

* S - steel, C - concrete

Figure 2. Cantilever column - fibers of the cross-section



Concrete strength was defined according to experimental data for analyzed viaduct (see Figure 6) and the shape of the diagram was defined according to Eurocode 2

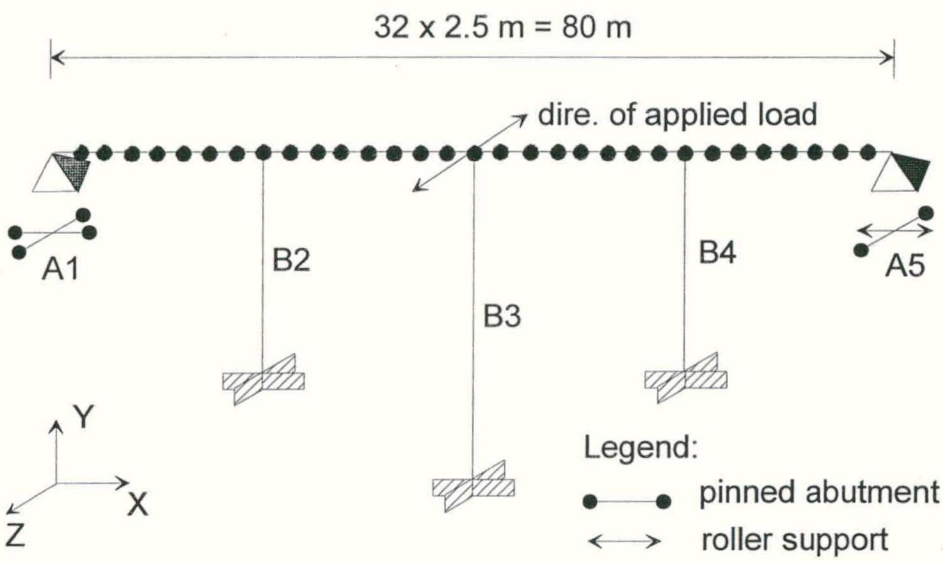
The tension strength of concrete was neglected

Steel properties were defined according to experimental data for analyzed viaduct and based on the limitation in Drain-3DX (it is not possible to model a flat part of the diagram after yielding)

Figure 3. Cantilever column - material properties

Viaduct

The second analyzed structure was a model of a four-span single-column-bent viaduct (Figure 4 and Figure 5). A scaling factor of 2.5 was used when defining the model of the real structure. Mass of the deck was modeled dividing the superstructure onto 32 segments of the equal lengths. A half of the column mass was added to the appropriate deck lumped mass, above each column. Since the superstructure was assumed to respond elastically to the strong ground motions, it was modeled with 32 elastic beam-column elements (type 17 in Drain-3DX). The abutments were modeled as infinitely rigid. Both abutments were pinned in the transverse direction of the viaduct. In the longitudinal direction of the viaduct left abutment was pinned and at the right abutment superstructure could move freely. Columns were pinned at the level of superstructure and fixed to their footings. Rotations of footings were neglected when modeling the viaduct.



Masses [t]

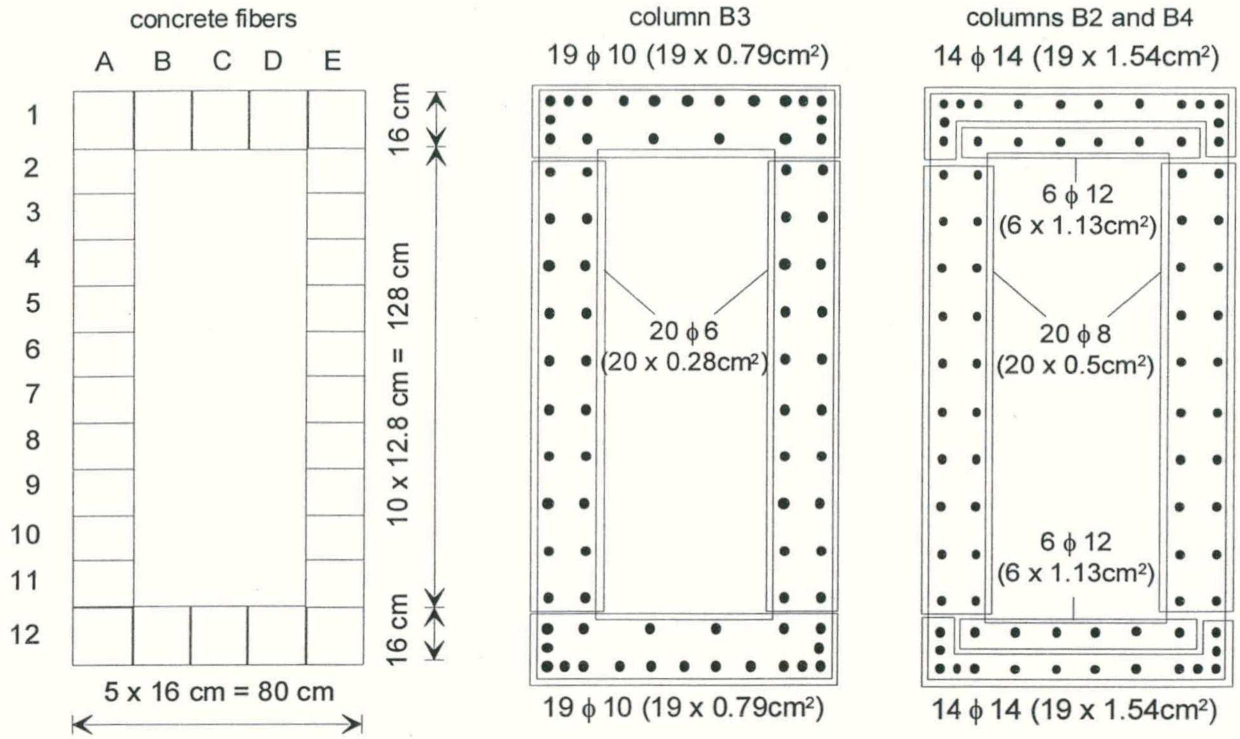
all (except at the top of B2, B3, B4)	8.15
B2	12.90
B3	15.28
B4	12.90

Superstructure properties

Area A = 1.1 m^2
Moment of inertia around Y-axis Iy = 2.26 m^4
Moment of inertia around Z-axis Iz = 0.135m^4
Modulus of elasticity E = 25 GPa

Abutment properties
Abutments were modeled as infinitely rigid.

Figure 4. Model of the viaduct



Neto area of concrete fibers was taken into account (area of steel fibers was subtracted)

Each bar was modeled as a single fiber.

Figure 5. Column properties

columns B2 and B4 *column B3*

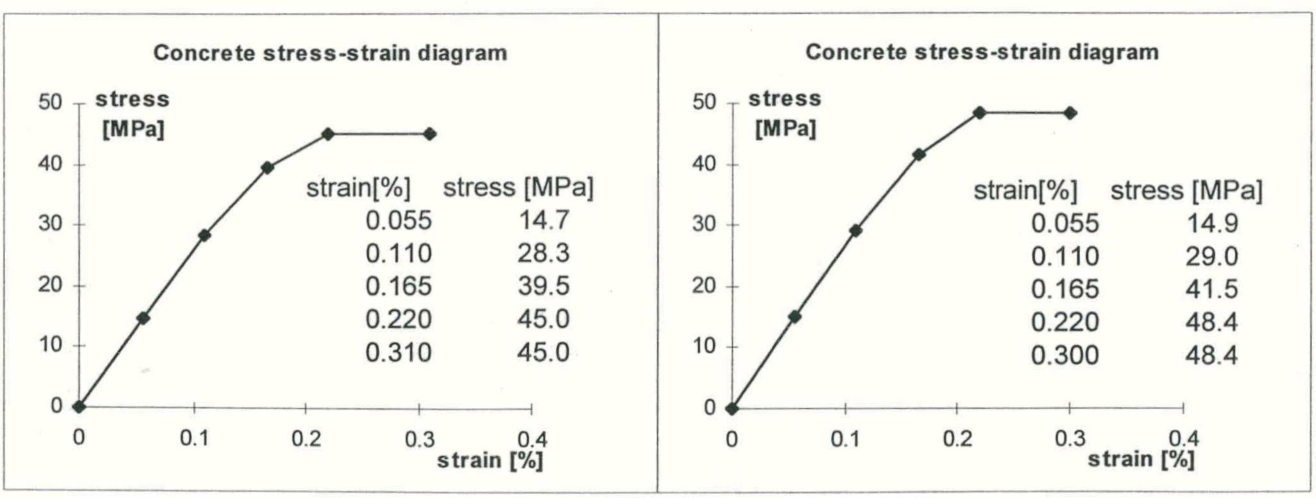
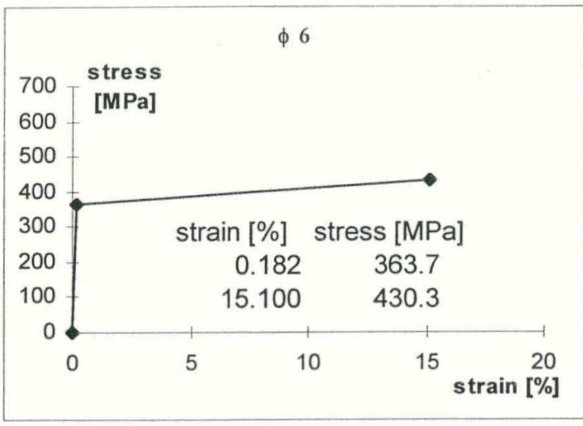
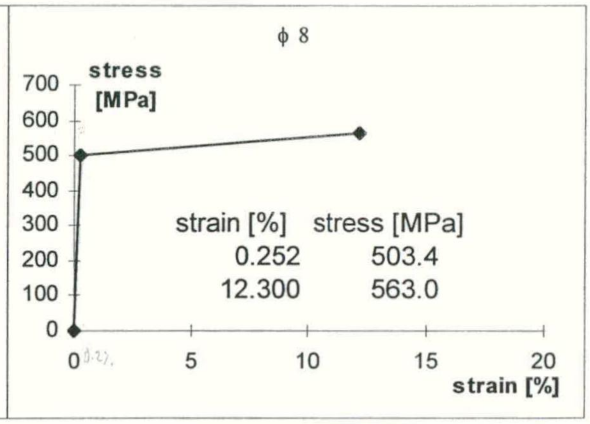
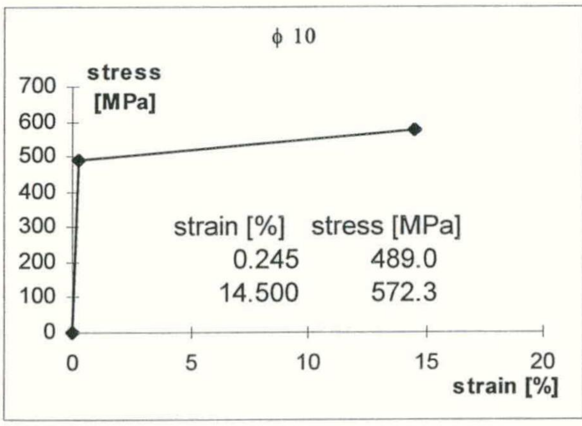
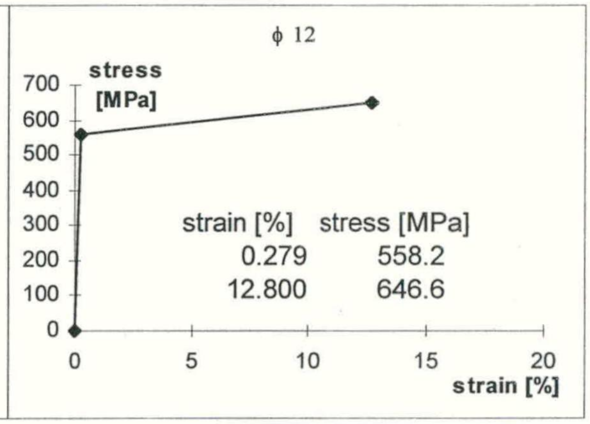


Figure 6. Concrete properties









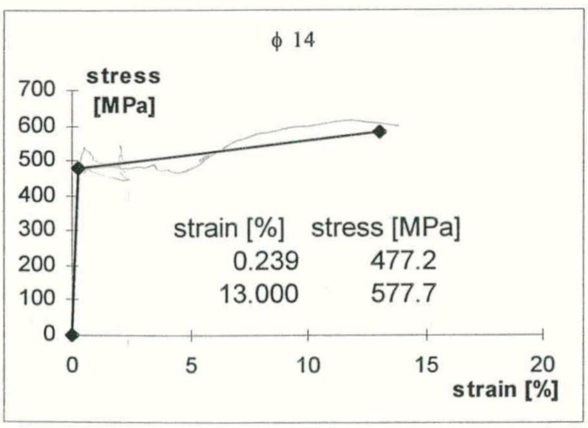


Figure 7. Steel properties

3 Vertical and seismic load

Cantilever column

A vertical force of 800 kN as well as a horizontal impulse load presented in Figure 8 were applied at the top of the cantilever column. When calculating the response, a constant time step of 0.002 s was selected.

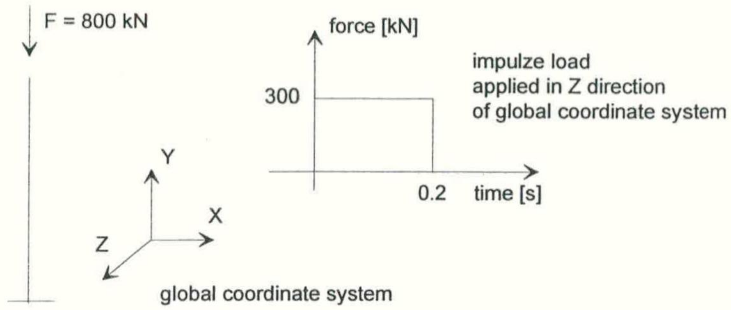


Figure 8. Cantilever column - Load

Viaduct

A vertical force of 1700kN was applied at the top of each column of the viaduct. Seismic load was defined with the generated earthquake record, originally used in the experimental studies. Since the analyzed viaduct is a model of the real viaduct (in scale 1 : 2.5), the earthquake load was also appropriately scaled. The duration of the earthquake record was reduced 2.5 times and acceleration was enlarged 6.25 times (see Pinto A. V. 1996 and Figure 9). Therefore the duration was 4 sec and peak acceleration was 0.875g. Earthquake load was applied in the transverse direction of the viaduct. When calculating the response a constant time step of 0.0005 s was chosen.

For both analyzed structures a 1.6% mass proportional damping was defined, since the same value was used in the pseudo-dynamic test of the viaduct.

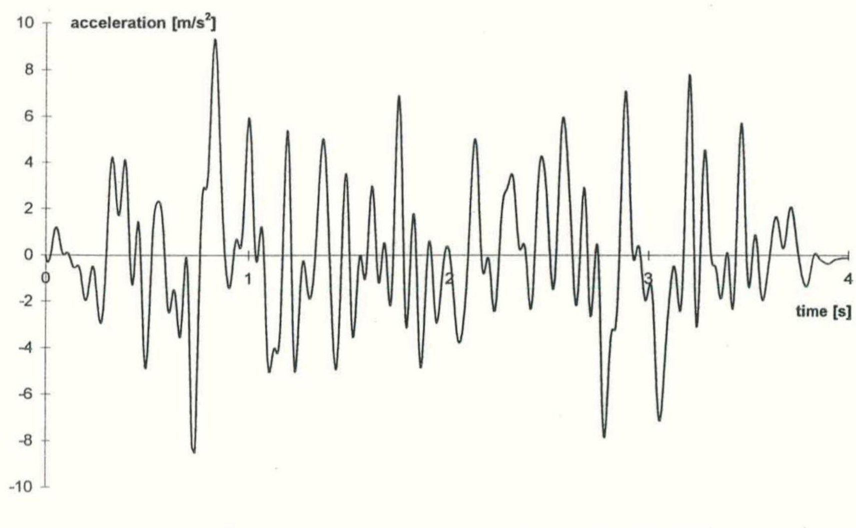


Figure 9. Generated earthquake record

4 Analysis of results

4.1 Introduction

When modeling columns with fiber beam-column element (type 15 in the Drain-3DX), columns are divided into certain number of segments. Each segment is assumed to have constant cross-section properties. Cross-sections are further divided into certain number of fibers. In reinforced concrete columns at least two types of fibers have to be defined, one for concrete and one for steel fibers. Each fiber type is described with related steel or concrete stress-strain relationship. It is assumed, within the Drain-3DX, that plane cross-sections remain plain after deforming. Therefore the bond slip is assumed to be zero within the body of the element. However, the bond slip in column-to-footing or beam-to-column connections can be taken into account defining the connection hinges at the end of an element.

When analyzing the response of structures, the bond-slip in the column-to-footing connections was neglected, first. Results are presented in section 4.2. After that, it was also taken into account. Since the data for modeling the bond slip were difficult to define, only the qualitative influence of this parameter was analyzed (see section 4.3).

4.2 Response considering columns with no bond-slip in the connection hinges

In the analyzed structures each column was modeled like a single fiber beam-column element. Each element was divided into different number of segments. The segment length was varied. Since the choice of the number of segments and the choice of their lengths were found the most important parameters influencing the response, they are presented in the next subsection in details.

Each column cross-section was divided into certain number of steel and concrete fibers. Longitudinal reinforcement of the cantilever column was modeled with four fibers (see Figure 2). In viaduct, each reinforcing bar of columns was modeled as a single fiber (see Figure 5). Number, positions and areas of steel fibers were kept constant. Properties of concrete fibers were varied. Initially a simple mesh of concrete fibers was chosen (see Figures 2 and 5). After that, the number of concrete fibers near the edge of the cross-section was increased. Since these changes of the model had negligible influence on the response of the structures, relevant results are not presented.

4.2.1 Influence of the number of segments and different segment lengths

In the Drain-3DX a constant stiffness along each segment is assumed. This is a very simplified solution which demands very careful use of the element 15 when modeling columns. The calculation of the stiffness of the segment is based on the deformations of the cross-section at the center of the segment (based on the related curvature). Since the slope of the curvature diagram changes very fast along the zone of the column

plastic hinge, changes of stiffness are also very large. Therefore, a response can be very sensitive to the choice of the number of element segments and the choice of the segment lengths.

Cantilever column

The influence of the number of segments and their lengths on the response will be illustrated on the example of the cantilever column, first. The column was modeled with a single element type 15, which was divided into one, two, four and eight segments of the equal length, respectively (see Figure 10). The cross-section type of all segments was fiber, with the same steel and concrete fibers (see Figure 2).

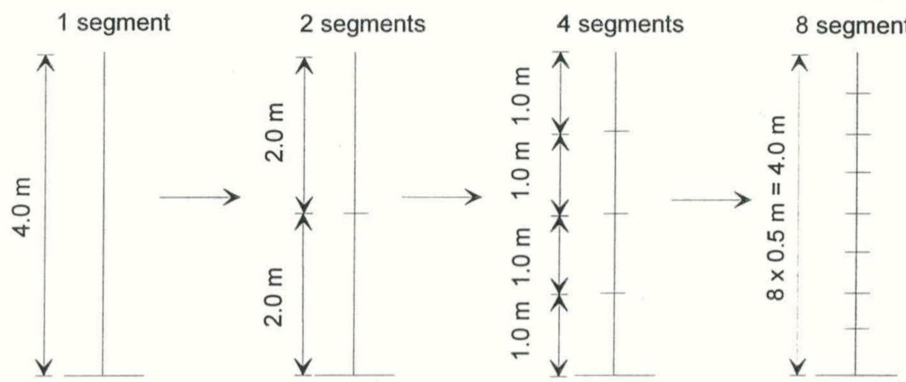


Figure 10. Models of cantilever column with different number of fiber segments

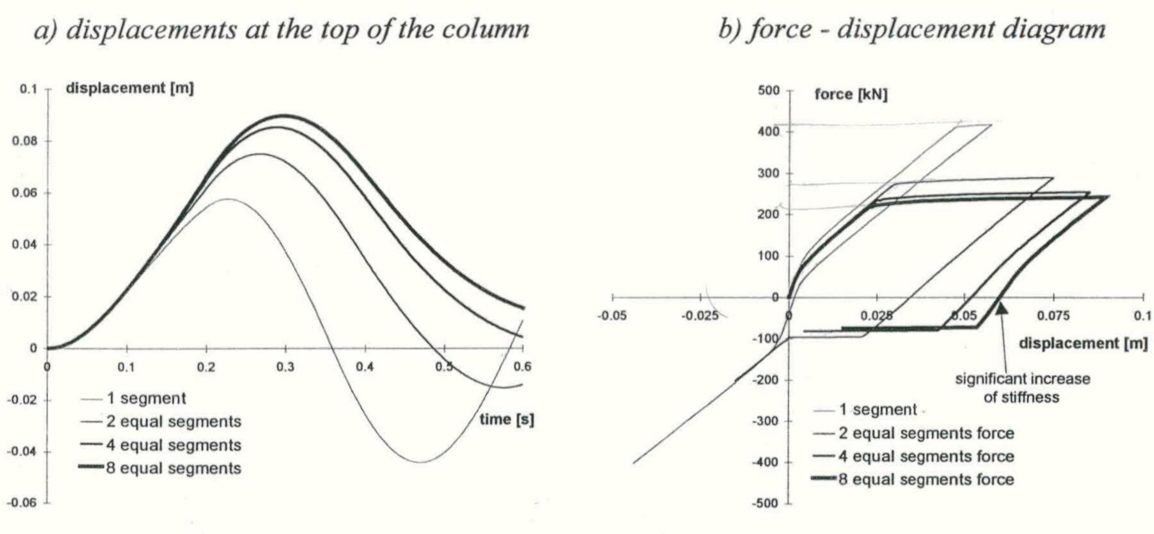
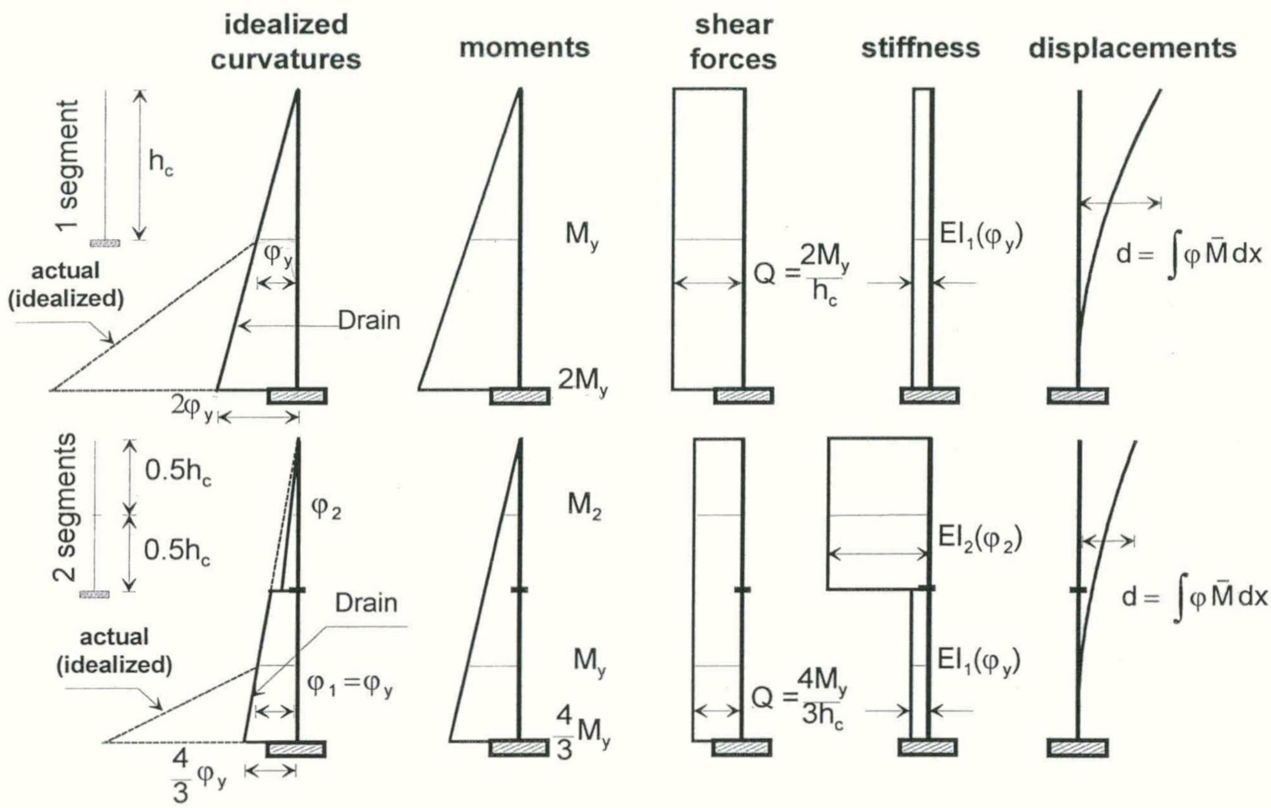


Figure 11. Response of the cantilever column modeled with different number of fiber segments

Since the program assumes constant cross-section properties along the segment (based on the cross-section properties at the center of each segment), number of segments along the element has significant influence on both, forces and displacements (see Figure 11). When column is modeled with only one segment, the ductility demand is very small, and the response is practically elastic. If the column is modeled

with larger number of segments, ductility demand increases, and the yielding shear force and yielding displacement decrease. The explanation is presented in Figure 12*, where the values of yielding shear forces and yielding displacements for two column models (with one and two segments) are analyzed.



Legend: φ_y - yielding curvature at the center of bottom segment, M_y - yielding moment at the center of bottom segment, Q - shear force, EI - stiffness, d - displacement h_c - column height, $\bar{M}$ - virtual moment

Figure 12. Yielding of column, which is modeled with one and two segments, respectively

When the column is modeled with one segment only, and when the yielding of column occurs, the yielding moment M_y is reached at the center of the column. Therefore, a moment at the level of footing is equal to double yielding moment ($M = 2M_y$). In the same column, which is modeled with two equal segments, yielding moment is reached at the center of the bottom segment at the level of $3/4h_c$ from the column top (h_c is the total column height). Therefore, the moment at the level of column footing is equal to $4/3M_y$. Since the shear force is equal to the moment at the column base divided with the column height ($Q = M/h_c$), yielding shear force Q_y in the column modeled with one segment is 1.5 times larger than the yielding shear force in the column modeled with two segments.

* All the presented quantities are related to the time step immediately after yielding of column is occurred, and where the stiffness of column is changed.

The yielding displacements obtained with these two models are different, too. Yielding displacement in column, modeled with one segment is larger because the curvatures along the whole element are larger than in the case when column is modeled with two segments.

The yielding displacement and yielding shear force, which are obtained using models with two, four and eight segments of equal length, are not so drastically different as in the case which is analyzed in the Figure 12. The main reason is that the levels of the centers of bottom segments (where yielding is occurred) are not so significantly different as in the previous case (see Table 1).

Table 1. Curvatures, moments and shear forces at the base of the column, when it is modeled with different number of segments

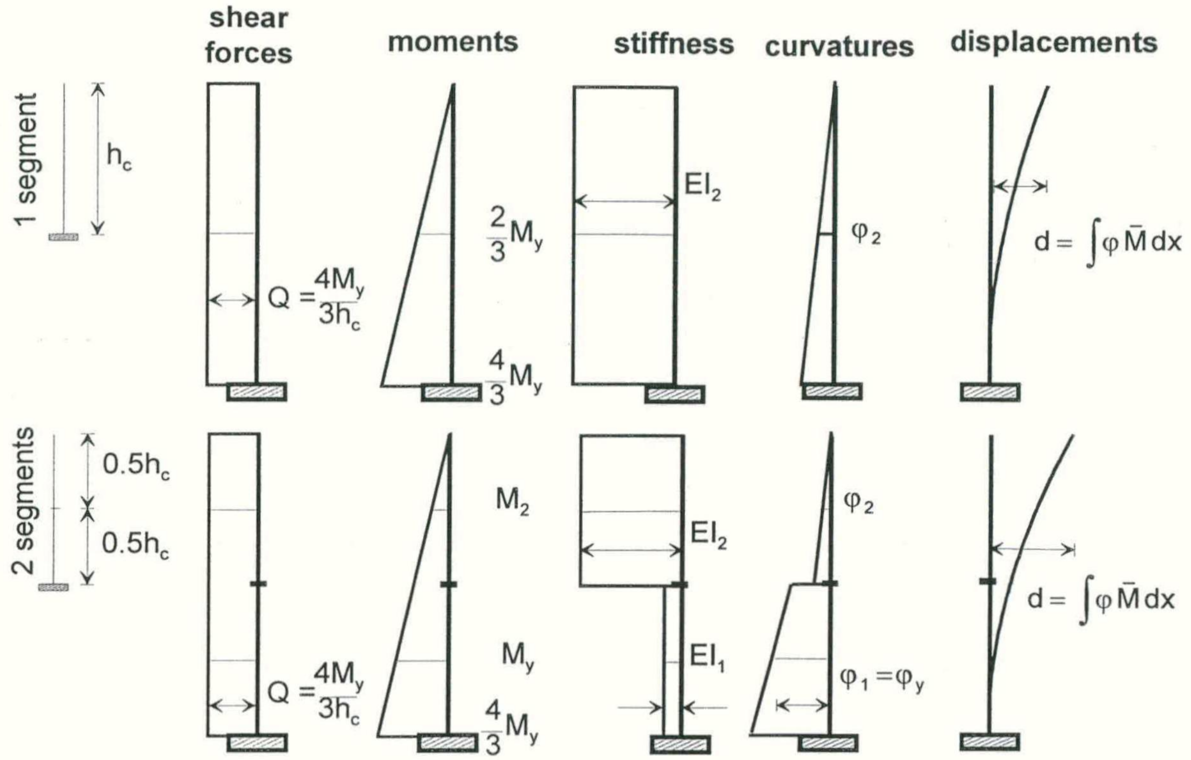
	1 segment	*2 segments*	*4 segments*	*8 segments*
h_{cbs}	$1/2\ h_c$	$3/4\ h_c$	$7/8\ h_c$	$15/16\ h_c$
φ_b	$2\ \varphi_y$	$4/3\ \varphi_y$	$8/7\ \varphi_y$	$16/15\ \varphi_y$
M_b	$2\ M_y$	$4/3\ M_y$	$8/7\ M_y$	$16/15\ M_y$
Q_c	$2\ M_y/h_c$	$4/3\ M_y/h_c$	$8/7\ M_y/h_c$	$16/15\ M_y/h_c$

h_{cbs} - distance of the center of the bottom segment from the top of the column; φ_b - curvature at the base of the column; M_b - moment at the base of the column; Q_c - shear force in the column; h_c - height of the column; φ_y - yileding curvature; M_y - yielding moment

The ultimate displacement in the column modeled with only one segment is significantly smaller compared to those obtained with other models. Explanation is presented in the Figure 13, where the response of two models (with one and two segments) under the equal shear force is analyzed. It is assumed that this shear force Q_c is equal to the yielding shear force of the model with two segments ($Q_c = 4/3\ M_y/h_c$). Therefore, in both models, the moment at the column base is $M_b = 4/3 M_y$.

In the model with only one segment, there is no yielding in the column, because moment at the center of segment is less than yielding moment ($M_{cs} = 2/3\ M_y$)** . At the same time, in the second model, yielding is reached at the bottom segment, since the moment at the center of segment is equal to the yielding moment $M_{csb} = M_y$. Consequently, stiffness of the bottom half of the column is significantly smaller than that in the model with only one segment. Therefore, the curvatures at the bottom half of the column are larger, and consequently, displacement at the top of the column is also larger.

** In the program Drain 3DX, constant cross-section properties are assumed along the whole segment. They are determined based on the cross-section properties at the segment center.



Legend: φ_y - yielding curvature, M_y - yielding moment, Q - shear force, EI - stiffness, d - displacement
h_c - column height, $\bar{M}$ - virtual moment

Figure 13. Response of the two models (with one and two segments) under the same lateral load

Based on the previous observations, it can be concluded that the proper modeling of the plastic hinge zone is extremely important for the good estimation of the column response. In order, to get the better estimation of the column response and to reduce the computing time it is necessary to model a column with the segments of different lengths, and to reduce the length of segments near the footing.

Therefore, new models (see Figure 14) for cantilever column were defined. It was assumed that the length of the plastic hinge in the column was equal to 10% of the column height (h_{ph} = 40 cm or 0.5D, where D is the section depth). First, the column was divided on two fiber segments (Figure 14a). The length of the bottom segment was equal to the length of the plastic hinge ($0.1h_c$, h_c is column height), and the rest of the column was modeled as one fiber segment. Then, column was divided on three fiber segments (Figure 14b). Plastic hinge was modeled in the same way as before, and the rest of the column was divided onto the two fiber segments of the same length (0.45 h_c). Since the large damage of the column can be expected in the zone of the plastic hinge only, the third model was defined (Figure 14c). Plastic hinge (length $0.1h_c$) was modeled with a fiber segment and the rest of the column with one elastic segment. Response obtained with these three models is presented in the Figures 15 and 16.

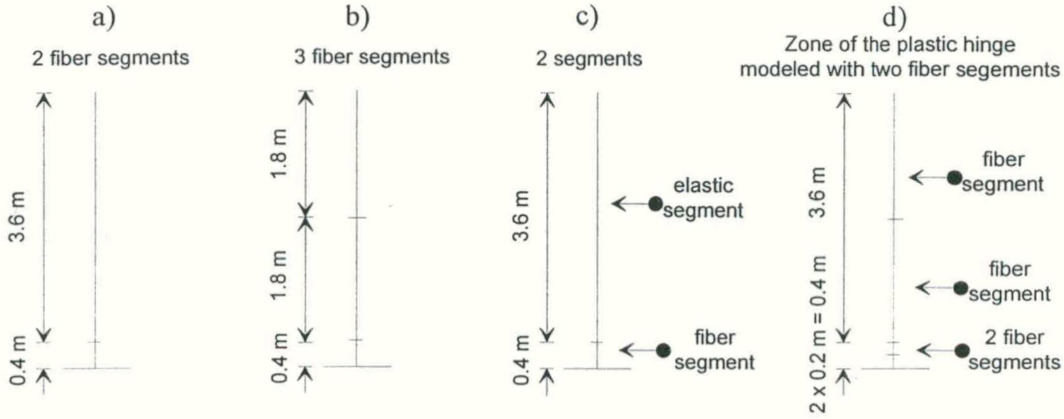


Figure 14. Cantilever column modeled with the segments of the variable length

a) displacements at the top of the column *b) force - displacement diagram*

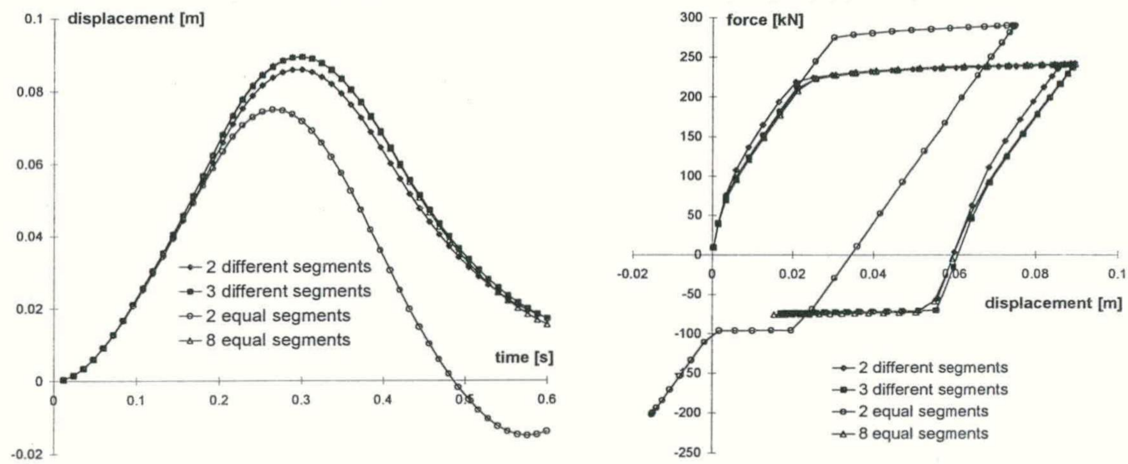


Figure 15. Response of cantilever column modeled with fiber segments of variable length

a) displacements at the top of the column *b) force - displacement diagram*

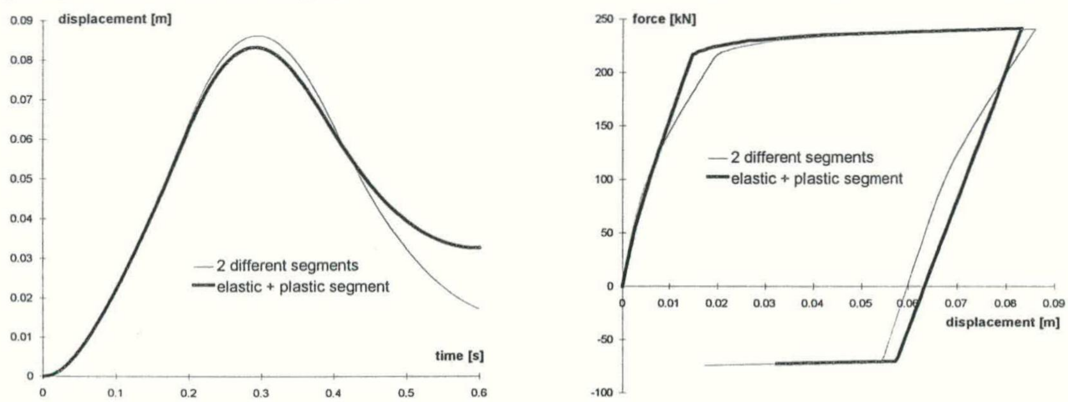


Figure 16. Response of cantilever column modeled with the fiber segments only and with mixed elastic and fiber segments

When column was modeled with only two fiber segments of different length, where the region of the plastic hinge was represented with one fiber segment, the response was very similar to that when eight fiber

segments of the equal length were selected. When column was modeled with three different segments, response was practically the same as in the case of eight segments of the equal length.

If the plastic hinge was modeled with one fiber segment and the rest of the column with the elastic segment, response was similar to that, when only fiber segments were selected. Yielding force and displacement as well as the maximum displacement were very similar in both cases. However, larger differences between displacements of these two models were observed when column was unloaded.

Since it was found that modeling of the plastic hinge zone had significant influence on response, another model of column was defined. It was similar to the model with three different fiber segments (see Figure 14d), except for the plastic hinge zone, which was modeled in different way (with two fiber segments). A response obtained with this model was essentially the same as in the case of three fiber segments of different length (see Figure 17).

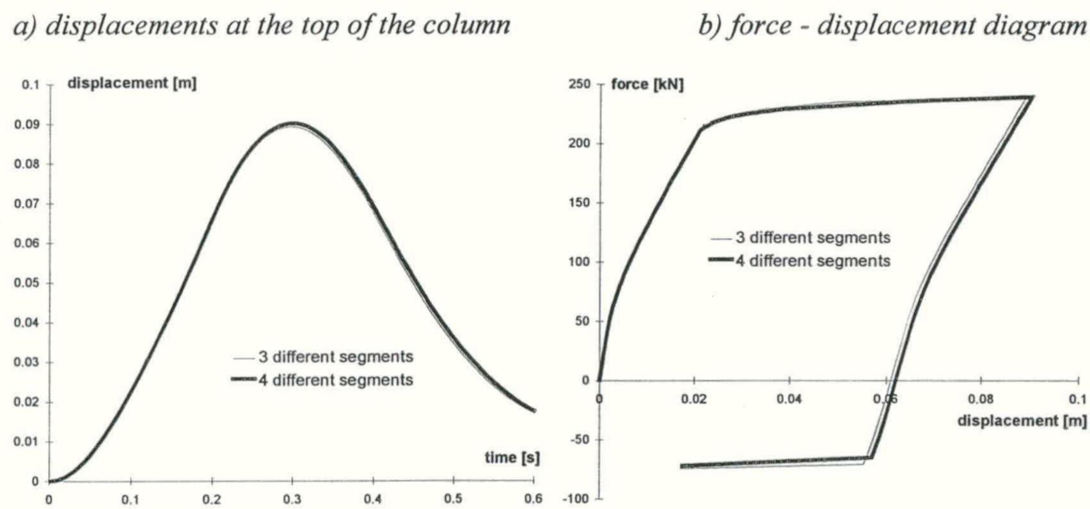


Figure 17. Response of cantilever column when plastic hinge zone is modeled with two fiber segments

It can be concluded that fiber segments of different lengths are very appropriate for modeling columns with fiber element type 15. The estimated plastic hinge zone has to be represented with at least one fiber segment and the rest of the column with one or two fiber or elastic segments. The important problem, however, is how to assume the length of the plastic hinge properly.

Viaduct

Based on previous conclusions, columns of the viaduct were also modeled in several different ways. Response was compared with the experimental results. Analytical results matched the experimental results the best (Figures 18, 19) when columns were modeled with two segments of different lengths. Both, the plastic hinge zone of each column as well as the rest of the column were modeled with one fiber segment. The length of the plastic hinge observed within the experiment was considered. In the short columns (B2

and B4) this length was 0.5 m. It was similar to the value of 0.59 m, which was obtained with the well known formula $L_h = 0.08L + 0.022d_sf_y$ (L_h is the length of the plastic hinge, L is the length of the column expressed in m, d_s is the diameter of the longitudinal reinforcing bars expressed in m, and f_y is the yielding stress of the longitudinal reinforcing bars expressed in MPa). However, in the column B3 the experimentally observed length was significantly smaller than the estimated one. Experimental length of the plastic hinge was only 0.28 m, while the formula gives the length of 0.78 m.

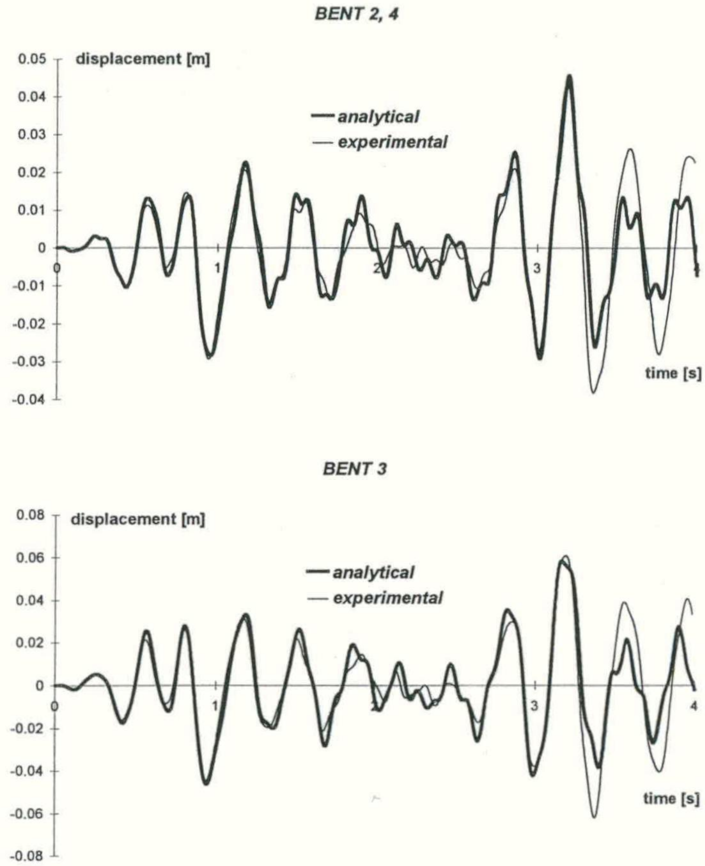


Figure 18. Displacement at the top of the columns over the time

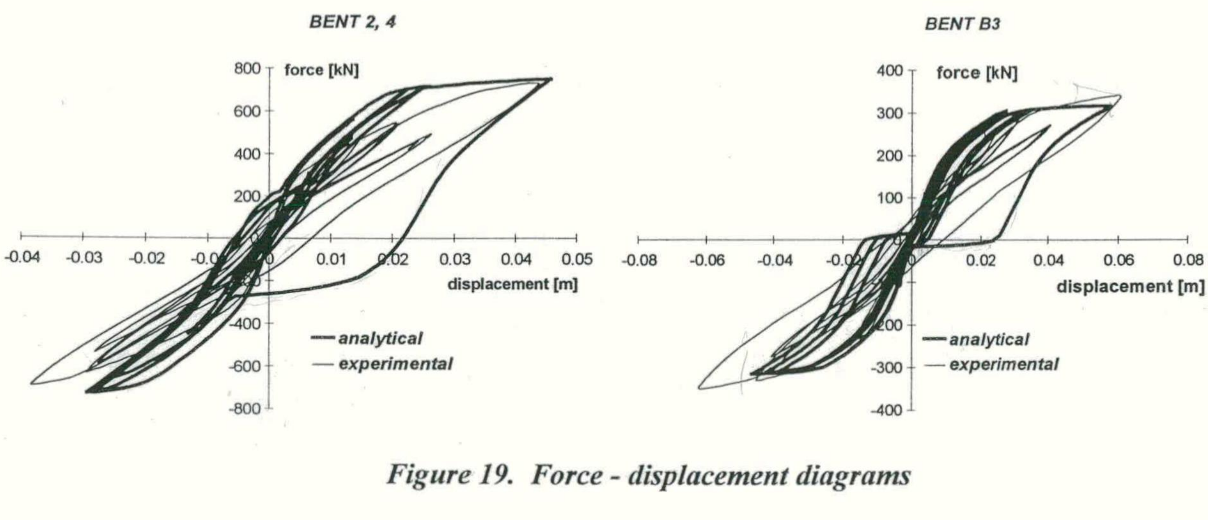


Figure 19. Force - displacement diagrams

The global response obtained with this model is relatively good. The maximum values of displacements and shear forces (Table 2) are similar to those obtained with the experiment. The displacement time history is satisfactory, too (Figure 18). However, the force-displacement relationship is significantly different from that obtained within the experiment (Figure 19). The differences are especially large after maximum displacements at the top of columns are reached and columns are in the phase of unloading. In the experimental results significant decrease of stiffness on column unloading can be observed. This stiffness is considerably smaller than the stiffness corresponding to cracked cross-section, where the yielding of reinforcement occurs. The analytical response shows the opposite trend. After certain point the stiffness gradually increases, whereas the yielding of the reinforcement in compression occurs. The same trend is observed in the analysis of the cantilever column (see Figure 20 in the next subsection), when it was modeled with more than one segment.

Table 2. Maximum values of shear forces and displacements

	experimental results		*analytical results*	
	B2, B4	**B3**	**B2, B4**	**B3**
max. shear force [kN]	730	340	760	350
max. displacement [cm]	4.4	5.9	4.4	6.2

Analysis of differences between the experimental and analytical results

Since the experimental and analytical results were so different, this phenomenon was additionally investigated on the example of cantilever column. It was found that the increase of stiffness in the phase of the column unloading started at the same time when the sign of the curvature of the column top segment is changed (Figure 21). At this time the top segment of the column is uncracked, and the cracks start to open at the opposite edge of the cross-section than those in the bottom segment of the column. Also, the cracks in the bottom segment are closing and both, tension and compression reinforcement are in the elastic range. Therefore, the stiffness of the whole column increases. After the maximum displacement at the column top is reached, the shear force in column is gradually reduced over time, and after certain time the sign of shear force is changed. The loading of the column in the opposite direction starts. When load becomes large enough, the reinforcement of the bottom segment at the tension edge starts to yield again (Figure 22). Therefore, the stiffness of the column rapidly decreases. The cracks of the bottom segment are closing very quickly. Finally, when reloading of both segments occurs, analytically obtained stiffness become similar to the experimental one, again (see Figure 19).

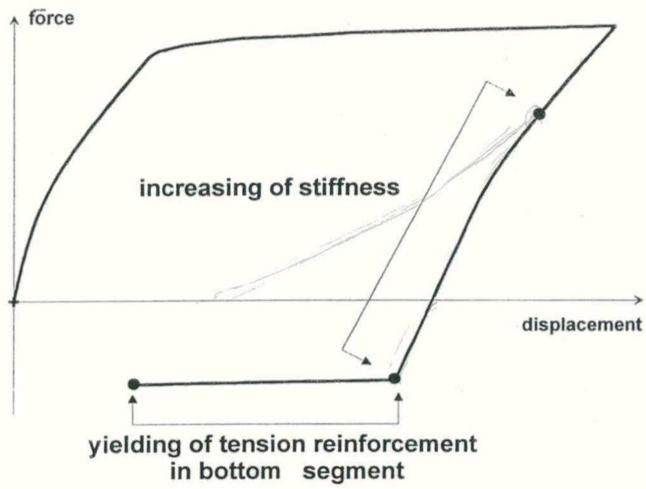


Figure 20. Response of the column modeled with two segments of different lengths

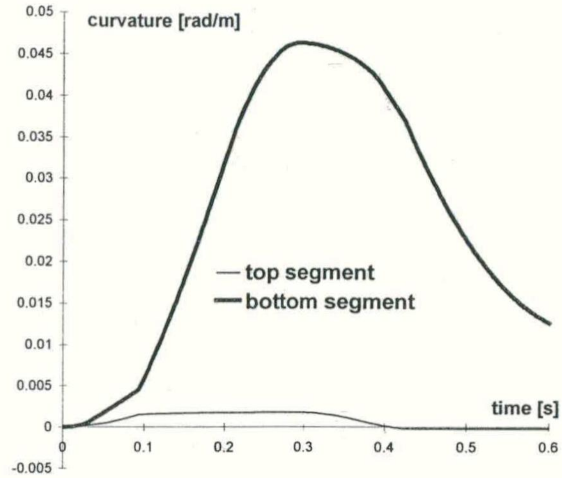


Figure 21. Curvature over the time in the top and bottom segment of the column

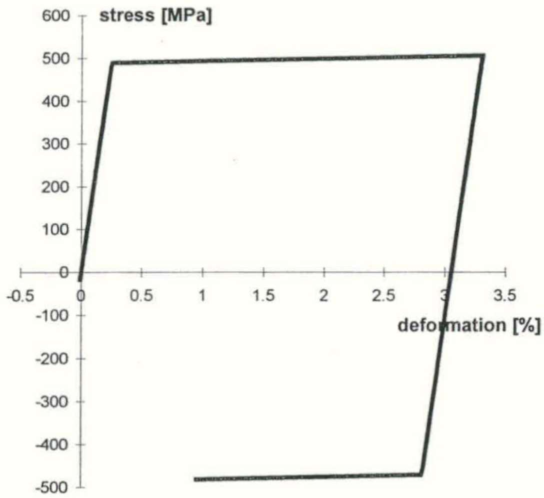


Figure 22. Stress-strain diagram of tension steel fibers (1 and 2) at the bottom segment of the column

Since the large difference between experimental and analytical force-displacement diagrams was obtained on unloading it can be concluded that in this range modeling of the column with the basic properties of the fiber element only, is not satisfactory. Since the experimental curves show the significant reduction of the stiffness on unloading, it is obvious that an important deterioration mechanism is not considered. This mechanism proved to be bond slip in the column-to-footing connections (see next section).

4.3 Response considering bond-slip in the connection hinges

It was believed that improved analytical model, where the pull-out properties of the column-to-footing connections were also considered, could result in the response which would fit the experimental results better. Therefore, this parameter was also taken into account. It was studied on the example of the viaduct, only.

The properties of the connection hinges were difficult to define. Moreover, a mistake in the used version of the program Drain 3DX was observed when defining more than one type of the connection hinge (see next section). Therefore, only the qualitative analysis of the bond slip was performed.

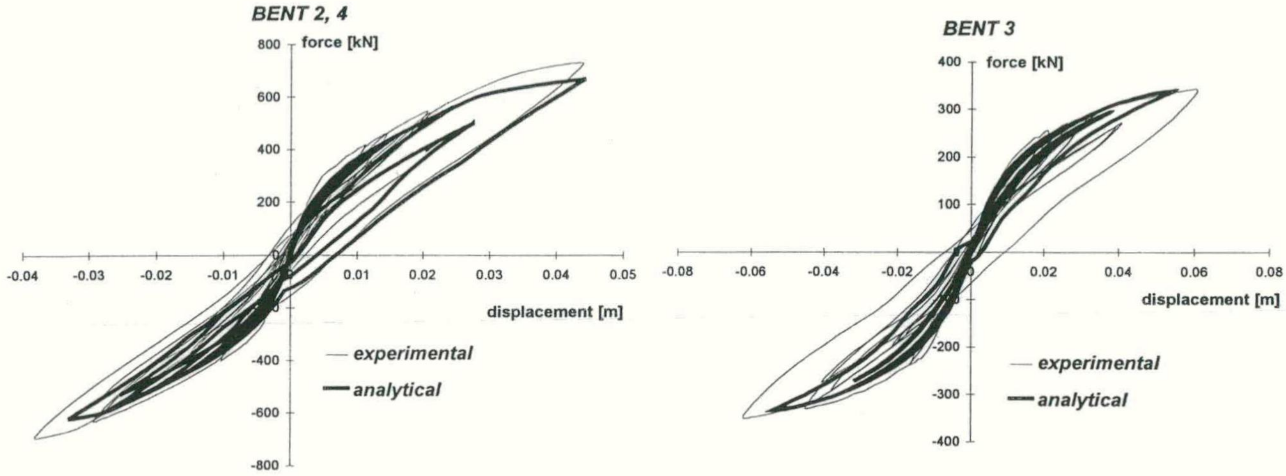


Figure 23. Force - displacement diagrams when bond slip is considered

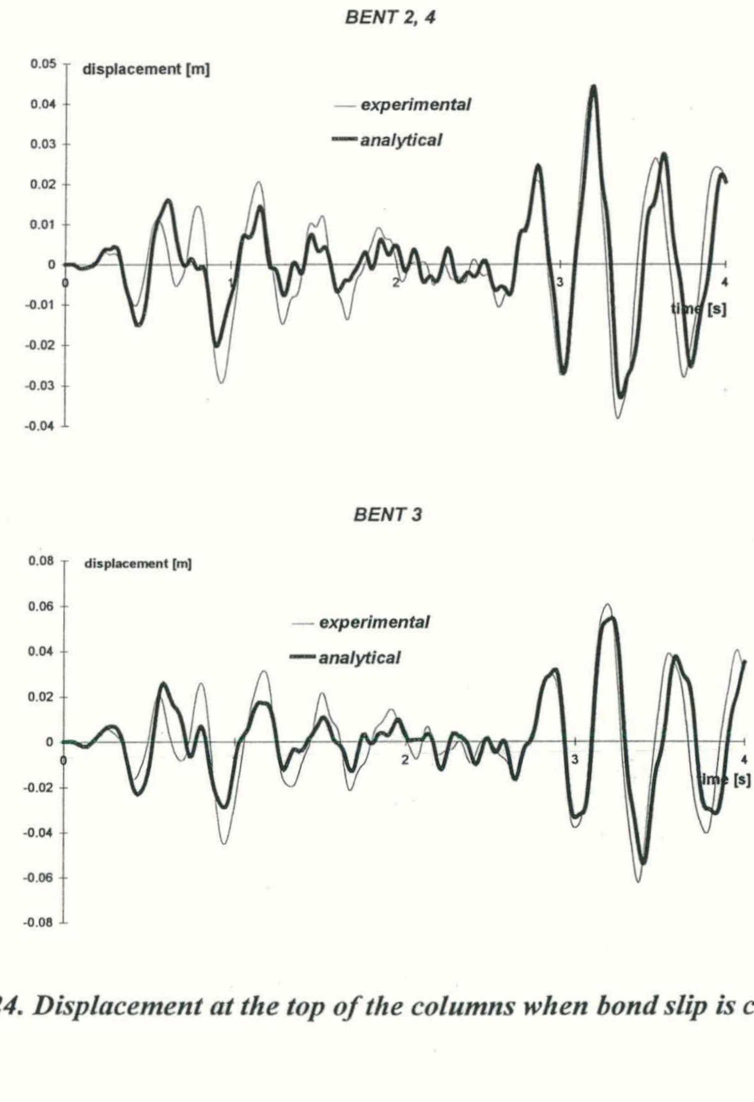


Figure 24. Displacement at the top of the columns when bond slip is considered

It is evident from Figure 23 that the response was better than in the case where the bond slip in the connection hinges was not considered. The stiffness on unloading is significantly reduced and it satisfactory match the experimental data, especially in the short columns. However, since the only one hinge type could be defined in the applied version of the program and average values for connection hinge properties of short and long columns were considered, the displacement history at the beginning did not fit the experimental data as good as without considering connection hinges (Figure 24). After the maximum displacement was reached, displacement history was significantly better than before.

4.4 Some notes about errors in the version of the Drain 3DX, used in the analysis

During the analysis of the cantilever column and the viaduct, two errors in Drain 3DX were observed:

1) According to the manual of the program, it is possible to define different values for stiffness degradation on unloading of concrete. This stiffness can be defined in the form of the unloading factor FU, which can take a value in the range from 0 to 1. However, this factor had no influence on the unloading stiffness. Although the value of 0.5 was chosen for the factor FU, the unloading was the same as in the case when the value of 1.0 would be selected (see Figure 25).
2) It was mentioned in the previous section that only one type of the connection hinge per structure can be defined. Since the analyzed viaduct includes two column types with significantly different reinforcement, it was necessary to define two types of the connection hinge, one for short and one for long column. However, when two hinge types had been selected, an error occurred in the program, in the static analysis of the viaduct, at the vertical load. Therefore, the dynamic analysis could not be performed and the program failed.

   The calculated rotations of the connection hinges were evidently wrong. Therefore, a dummy vertical load above the columns was applied. First type of the connection hinge was related to the short columns (B2 and B4), and second type of connection hinge was related to the long column (B3). While in the connection hinges of short columns program calculated zero rotations around Z-Z axis, the hinge rotation was huge (equal to the initial modulus of elasticity of steel, about 2×10^8) in the long column.

   After that the model of the viaduct was changed. The first type of the connection hinge was related to the long column, and the second type of the connection hinge to the short columns. Again zero vertical load above the columns was applied. In this model huge rotations were calculated in the connection hinges of the short columns, while in the long column a zero rotation was obtained.

Since the structure of the program Drain 3DX is relatively complex, the detected errors are not corrected for a now.

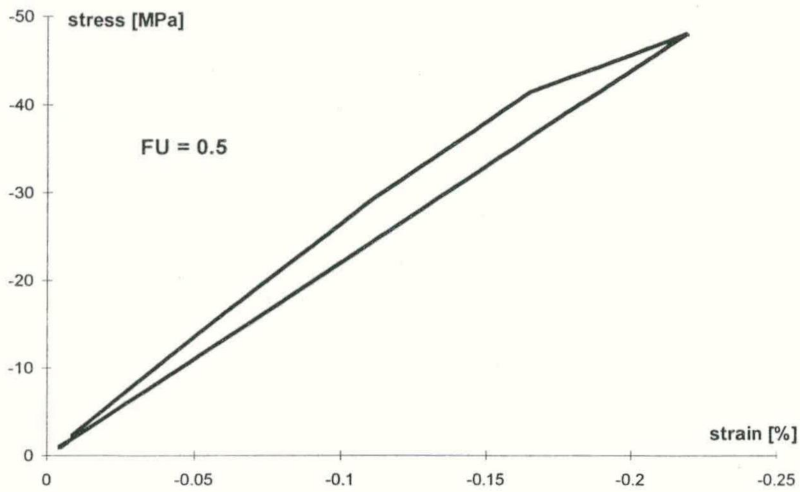


Figure 25. An example of stress-strain diagram of the concrete fiber

5 Conclusions

Theoretically, beam-column fiber element type 15 is a very precise and good tool for the non-linear analysis of the bridge piers. However, the practical use of the element showed some serious deficiencies of the element. It can be summarized that element type 15 is a very complex element, which is very difficult to control. The analysis of the data and the results is very demanding. For proper modeling of columns with this type of the element, the data, which are difficult to access or to estimate are necessary. Therefore, it can be concluded that simpler elements might be more efficient.

Based on the results presented in the previous sections, it is evident that global response of the structure, where some of the structural elements were modeled with element type 15, was fairly good. However, if the bond slip in the column-to-footing connections was not taken into account, hysteresis behavior was considerably different from that, which was obtained in the experimental investigations.

Although the element is very complex in general, the calculation of the element stiffness is very rough. Stiffness of the segment is based on the properties of the cross-section in the center of the segment. It is constantly distributed over the segment. Therefore, a precise modeling of the plastic hinge zone is necessary. This is however, the next inefficiency of the element, since the length of this zone in the majority of the cases is not known and it can be estimated only, by different empirical formulas. On the other hand, if some, more adequate stiffness distribution over the segment would be chosen, the complexity of the element would be additionally enlarged.

Since the element is very complicated, some errors in the original source were made.

6 REFERENCES

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